Graphs encoding the structure of a finite group

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The idea to associate a graph to a group goes back to the XIX century when Cayley introduced a graph, today known as the Cayley graph, with the aim to grasp information about an abstract group studying the invariants of the graph associated with it. This research area produced many interesting results in group theory, and in turn it also allowed to exhibit several examples of graphs with specific properties.

Among all graphs that can be considered to study a finite group, one can examine graphs whose vertices are the elements of the group and whose edges encode the structure of the group itself. More precisely, if G is a finite group and $\mathcal{B}$ is a class of groups, the $\mathcal{B}$-graph associated with G, denoted by $\Gamma_{\mathcal{B}}(G)$, is a simple and undirected graph whose vertices are the elements of G and there is an edge between two elements x and y of G if the subgroup generated by x and y is a $\mathcal{B}$-group. Many properties of a finite group can be detected looking at the invariants of its $\mathcal{B}$-graph. For instance, one can study the completeness of $\Gamma_{\mathcal{B}}(G)$ to understand how far the group G is from being a $\mathcal{B}$-group. A new trend deals with studying the properties of the neighborhood $I_{\mathcal{B}}(x)$ of a vertex x in $\Gamma_{\mathcal{B}}(G)$, that is, the set of all y in G such that x and y generate a $\mathcal{B}$-group. Even though $I_{\mathcal{B}}(x)$ is not a subgroup of G in general, it can happen that the features of a single neighborhood could affect the structure of the whole group G.

The aim of this talk is to present recent results on this topic which involve the analysis of numerical and combinatorial invariants of neighborhoods in a $\mathcal{B}$-graph.