

# $\kappa$ -EXISTENTIALLY CLOSED GROUPS, CENTRALIZERS AND AUTOMORPHISMS

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Let  $\kappa$  be an infinite cardinal. A group  $G$  is called a  $\kappa$ -*existentially closed group* if every system of less than  $\kappa$  many equations and inequations with coefficients in  $G$  which has a solution in a group containing  $G$  already has a solution in  $G$ . The definition of a  $\kappa$ -existentially closed group was first given by W. R. Scott [4], inspired by the concept of algebraically closed fields. We will talk about recent developments in the class of  $\kappa$ -existentially closed groups, existence and uniqueness, centralizers of subgroups generated by fewer than  $\kappa$  elements see [1]. Finally we mention automorphism groups of  $\kappa$ -existentially closed groups of cardinality  $\kappa$  see [2]. In 2018 together with O. H. Kegel [3] we proved the following:

**Theorem.** *Let  $\kappa$  be an uncountable cardinal. Any two  $\kappa$ -existentially closed groups of cardinality  $\kappa$  are isomorphic.*

If we remove the cardinality condition, then we show that uniqueness is no more true, see [1].

We aim to review some recent results concerning  $\kappa$ -existentially closed groups obtained recently by Otto H. Kegel, Mattia Brescia, Kivanç Ersoy, Burak Kaya, Patrizia Longobardi and Mercedes Maj. We will also present some open questions.

## References

- [1] Mattia Brescia, Kivanç Ersoy and Mahmut Kuzucuoğlu,  *$\kappa$ -Existentially closed groups: centralizers and maximal subgroups*. Submitted.
- [2] Burak Kaya, Mahmut Kuzucuoğlu, Patrizia Longobardi, Mercedes Maj, *Limit Groups and Automorphisms of  $\kappa$ -existentially closed groups*, Submitted. arXiv:2409.00545
- [3] Otto H. Kegel and Mahmut Kuzucuoğlu,  *$\kappa$ -Existentially closed groups*, *J. Algebra*, **499**, (2018), 298–310.
- [4] William R. Scott, *Algebraically closed groups*, *Proc. Amer. Math. Soc.*, **2**, (1951), 118–121.